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A HYBRID k–ε TURBULENCE MODEL OF RECIRCULATING FLOW

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SUMMARY

This investigation deals with the modification of streamline curvature effects in the k–ε turbulence model for the case of recirculating flows. Based upon an idea that the modification of curvature effects in C_2 should not be made in regions where the streamline curvature is small, a hybrid k–ε model extended from the modification originally proposed by Srinivasan and Mongia is developed. A satisfactory agreement of model predictions with experimental data reveals that the hybrid k–ε model can perform better simulation of recirculating turbulent flows.

KEY WORDS Turbulence Recirculating flow

1. INTRODUCTION

The internal flow field in some engineering practices is highly turbulent and recirculating. Unfortunately, turbulence is one of the unsolved problems of today in the physical sciences. Many turbulence models incorporating phenomenological assumptions have been developed for the simulation of recirculating turbulent flows. The degree of success of a turbulence model depends on the nature and accuracy of the phenomenological assumption(s).

Turbulence models can be categorized in several ways. The following method is one often used in engineering practice.

(1) turbulence–viscosity models in which the length scale of turbulence is found by way of algebraic formulae (or zero-equation models)
(2) turbulence–viscosity models in which the length scale of turbulence is found from partial differential equations of transport (one- or two-equation models)
(3) models in which the shear stress itself is the dependent variable of a partial differential conservation equation (stress equation models).

These models have been described in several excellent review papers, e.g. References 1 and 2. Zero- and one-equation models have been successful in predicting simple flows but not in predicting complex flows such as flows with recirculation. Two-equation models, which still employ the Boussinesq eddy viscosity concept but are more complex than the aforementioned turbulence models, have since been used in many applications of engineering practice. Among the two-equation models, the k–ε model has been the most successful so far. However, it was found[3] that predictions using the k–ε model for flows with significant streamline curvature are only qualitatively correct. An attractive alternative is the use of higher-order models such as the algebraic stress model or the Reynolds stress model. In highly swirling and recirculating flows the

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improvement of flow field predictions using the algebraic stress model is not so pronounced in comparison with that using the k–ε model. For such flows the Reynolds stress model can, to some extent, improve the predictions of flow fields, but it greatly increases the computational complexity and time requirement. Furthermore, the fact remains that as the order of the turbulence model is increased, so the number of empirical parameters increases, and insufficient model validations lead to less generality of those empirical parameters in applications. From the engineer's point of view, a general method for predicting flow fields must comprise both a physical model which reflects the true nature of the flow, and an efficient mathematical apparatus which permits accurate and economical calculations. It is thus not clear whether higher-order models are more valuable than the k–ε model. For this reason the present study focuses on the k–ε model. It is known that both recirculating and swirling flows can generate significant streamline curvature. However, since swirling flows may possess spiral and recirculating motion simultaneously, which is a more complex phenomenon than purely recirculating flow, recirculating flows without swirl are therefore selected as the test problem here. However, the extension of this work to swirling flow will be examined later in another study.

In recognition of the less agreeable predictions of recirculating flow with the k–ε model, many modifications to the k–ε model have been developed. The one developed by Srinivasan and Mongia[4] was reported to be able to yield better predictions in the recirculation zone of the flow field and is therefore used for this study. The objective of the paper is first to investigate both improvements and drawbacks with the selected modified k–ε model and then to present a hybrid k–ε model which will give better predictions of flow fields.

2. MODIFICATION OF CURVATURE EFFECTS TO k–ε MODEL

An extensive review of the effects of streamline curvature on turbulence phenomena as well as modelling was given in References 1 and 3. The standard or original k–ε model describes the turbulence characteristics at any point in the flow field by a single length scale which is obtained from a hypothesis of isotropic turbulence structure, and relates the Reynolds stress to the rate of strain by two scaling parameters, i.e. the turbulent kinetic energy k and its dissipation rate ε, as

$$\mu_t = C_\mu \rho k^2/\varepsilon. \tag{1}$$

These two parameters are determined by their transport equations. A detailed study of the experimental results[3] reveals that the turbulence structure near the recirculation zone is quite anisotropic. This inference stems from the extra strain rates imposed on the strong recirculation or swirl zone, which would tend to increase both the velocity and the length scale of turbulence. When significant streamline curvatures are introduced into this kind of flow field, the standard k–ε model cannot adequately account for the enhanced turbulence diffusion caused by the extra strain rates associated with streamline curvature.

One popular way of accounting for curvature effects is to introduce corrections to the length-scale-determining ε-equation. The constant C_2 appearing in the ε-equation (see Table 1) is corrected as a function of the Richardson number, which is a measure of the extra strain rate due to streamline curvature.

Srinivasan and Mongia[4] further split the Richardson number into two parts—the swirl Richardson number and the curvature Richardson number—and corrected C_2 by

$$C_2 = 1{\cdot}92\,\exp(2\alpha_s Ri_s + 2\alpha_c Ri_c). \tag{2}$$

The swirl Richardson number Ri_s is set to zero for the present case of non-swirling flows, and the curvature Richardson number is defined as

Table I. Expressions corresponding to equation (4)

ϕ	Γ_ϕ	S_ϕ
1	0	0
u	μ_{eff}	$-\dfrac{\partial p}{\partial x}+\dfrac{\partial}{\partial x}\left(\mu_{eff}\dfrac{\partial u}{\partial x}\right)+\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\mu_{eff}\dfrac{\partial v}{\partial x}\right)$
v	μ_{eff}	$-\dfrac{\partial p}{\partial r}+\dfrac{\partial}{\partial x}\left(\mu_{eff}\dfrac{\partial u}{\partial r}\right)+\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\mu_{eff}\dfrac{\partial v}{\partial r}\right)-2\mu_{eff}\dfrac{v}{r^2}$
k	μ_{eff}/σ_k	$G_k-\rho\varepsilon$
ε	$\mu_{eff}/\sigma_\varepsilon$	$(C_1 G_k - C_2\rho\varepsilon)\varepsilon/k$

$$\mu_{eff}=\mu+\mu_t$$

$$G_k=\mu_{eff}\left\{2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial r}\right)^2+\left(\frac{v}{r}\right)^2\right]+\left(\frac{\partial u}{\partial r}+\frac{\partial v}{\partial x}\right)^2\right\}$$

$$Ri_c=\frac{uv}{u^2+v^2}\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)-\frac{\partial u}{\partial x}\right)\bigg/\left(\frac{1}{r}\frac{\partial}{\partial r}(ru)+\frac{\partial v}{\partial x}\right). \tag{3}$$

The value of C_2 is suggested to be in the range from 0·1 to 2·4 by Abujelala and Lilley[5] for swirling and recirculating flows. However, the optimum value of α_c can be arrived at by parametric studies. Srinivasan and Mongia[4] claimed that their modification to the k–ε model was capable of yielding reasonably satisfactory predictions, particularly in the recirculation zone. One major drawback of this modified k–ε model is that it overlooks the fact that the standard k–ε model works quite successfully for predictions of simple flows, and even simple flows possess some inherent streamline curvature albeit of smaller magnitude than in complex flows. We observe that a hybrid model, which permits the standard k–ε model to work in the small-streamline-curvature flow regions while allowing the modification to curvature effects in the turbulence model to work in the high-streamline-curvature flow regions, is expected to perform a better job than other modification methods.

3. NUMERICAL SOLUTION PROCEDURE

Calculation of the recirculating flow field in a sudden-expansion duct requires the simultaneous solution of the governing equations. The transport equations representing the conservation of mass, momentum, turbulent kinetic energy and its dissipation rate are cast into a general form of steady state and axisymmetric cylindrical co-ordinates:

$$\frac{\partial}{\partial x}(\rho u\phi)+\frac{1}{r}\frac{\partial}{\partial r}(\rho v r\phi)-\frac{\partial}{\partial x}\left(\Gamma_\phi\frac{\partial\phi}{\partial x}\right)-\frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\phi\frac{\partial\phi}{\partial r}\right)=S_\phi, \tag{4}$$

where ϕ is a general dependent variable. The corresponding expressions of Γ_ϕ and S_ϕ are given in Table I.

The finite volume method incorporated with the power-law scheme and SIMPLER algorithm[6] was employed to obtain the numerical solutions of the partial differential equations represented in the form of equation (4). The simultaneous and non-linear nature of the governing equations necessitates that special measures should be used to procure numerical stability (convergence). The commonly used measure is the successive underrelaxation method. The values of the underrelaxation factors were chosen from 0·5 to 1·0. There is no need to maintain the same value of the underrelaxation factor during the entire computation. The optimal value for the initial underrelaxation factor can only be found by experience and from numerical experiments for a given problem. The convergence criterion adopted in the present calculations was that the summation of the absolute values of the mass residual in the entire computational domain be less than 10^{-7}. The wall function treatment employed in the CHAMPION computer code[7] (see Appendix I) was introduced to bridge the wall layer to the fully turbulent region. The grid lay-out used for the calculation domain consisted of 36 × 28 non-uniformly distributed nodes, with a dense grid line concentration in the recirculation zone. Numerical tests showed that this non-uniform grid lay-out produced a nearly mesh-independent solution (with less than 0·1% change in reattachment length).

4. RESULTS AND DISCUSSION

Chaturvedi's experimental results[8] were first selected as the comparison basis for this study. The configuration of the sudden-expansion duct is sketched in Figure 1, with dimensions of $L = 2642$ mm, $r_a = 108$ mm and $R = 216$ mm. The Reynolds number of the mean flow at the inlet (r_a) was maintained at approximately 2×10^5. The expansion ratio (EPR), which is defined as

$$\mathrm{EPR} = R/r_a, \tag{5}$$

is 2·0 according to the given geometric configuration. The velocity measurements were made by both hot wire anemometer and Pitot tube. A constant velocity distribution with an almost zero level of turbulence was established by providing a suitable bellmouth entry.

No measurements in the inlet region were reported in Chaturvedi's experiments. A uniform mean velocity profile was specified at the inlet, and the inlet profiles for k and ε were given in the following empirical manner:

$$k_{in} = 0{\cdot}003 u_{in}^2, \tag{6}$$

$$\varepsilon_{in} = C_\mu k_{in}^{1{\cdot}5} / 0{\cdot}03 r_a. \tag{7}$$

Since the computational domain extends to an axial distance of 12·2 diameters of the inner duct, the outflow boundary condition is reasonably assumed to be fully developed in the calculation.

The predictions using the standard k–ε model provide a baseline with which to compare predictions using other modified k–ε models. The predicted distributions of the mean axial velocity and the turbulent kinetic energy at five different sections are shown and compared with the measured results of Chaturvedi[8] in Figure 2. Note that the velocity components and the

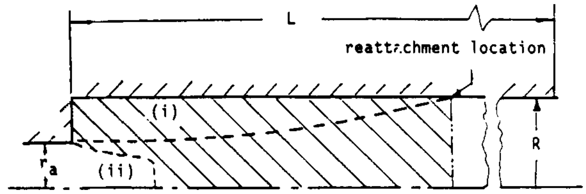

Figure 1. Schematic view of sudden-expansion duct: (i) recirculation zone; (ii) potential core

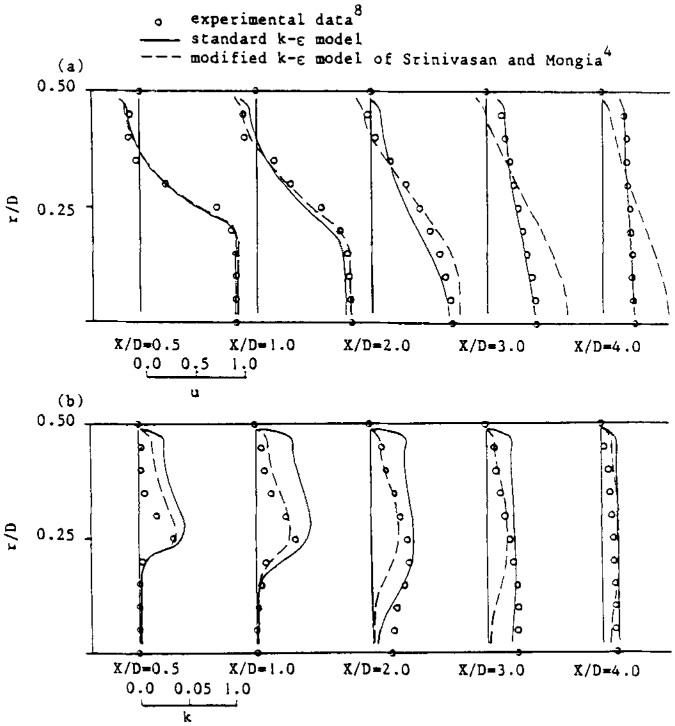

Figure 2. Comparison of predicted and measured[8] (a) axial velocities and (b) turbulent kinetic energy in test problem 1

turbulent kinetic energy shown in all figures are in dimensionless form and are non-dimensionalized by the reference quantities u_{in} and u_{in}^2 respectively. The contour of the curvature Richardson number within the sudden-expansion duct is plotted in Figure 3. The shaded regions represent streamlines with slightly negative values (of the order of -10^{-2}) of Ri_c. The higher values of Ri_c appear in the recirculation zone, as indicated in Figure 3. The highest value of Ri_c is about 0·3.

4.1. Modification proposed by Srinivasan and Mongia[4]

Before we apply the modification to the k–ε model proposed by Srinivasan and Mongia to the test problem, we have to determine the value of α_c appearing in equation (2). Figure 4 demonstrates the change of C_2 with Ri_c at four values of α_c. Here the minimum value of C_2 is limited to 0·1 according to the study of Abujelala and Lilley.[5] Since C_2 is an exponential function of Ri_c, the correction of C_2 approaches an asymptotic value when the parameter α_c moves to the more negative side, as depicted in Figure 4. Srinivasan and Mongia suggested that $\alpha_c = -2$ in their report.[4] Several different values of α_c were tried in the present test run, and among those, the best comparison with measured data was obtained for α_c between -2 and -3. For α_c approaching the more negative side, C_2 falls more sharply (see Figure 4), which implies that the flow regions with less streamline curvature will be much distorted. The value of α_c was therefore selected to be -2 in the following calculations.

The predicted distributions of the mean axial velocity and the turbulent kinetic energy are also presented in Figure 2. A comparison of the predicted results using two k–ε models with the

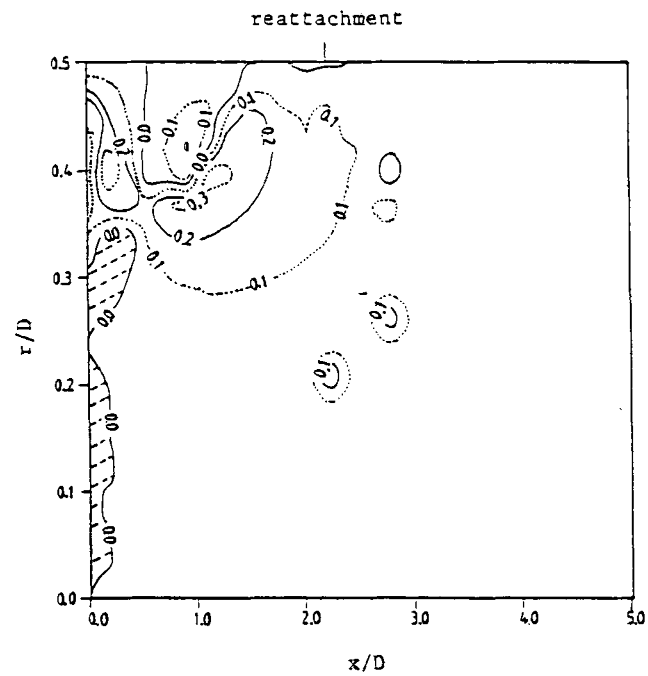

Figure 3. Contour of curvature Richardson number predicted by standard k–ε model in test problem 1

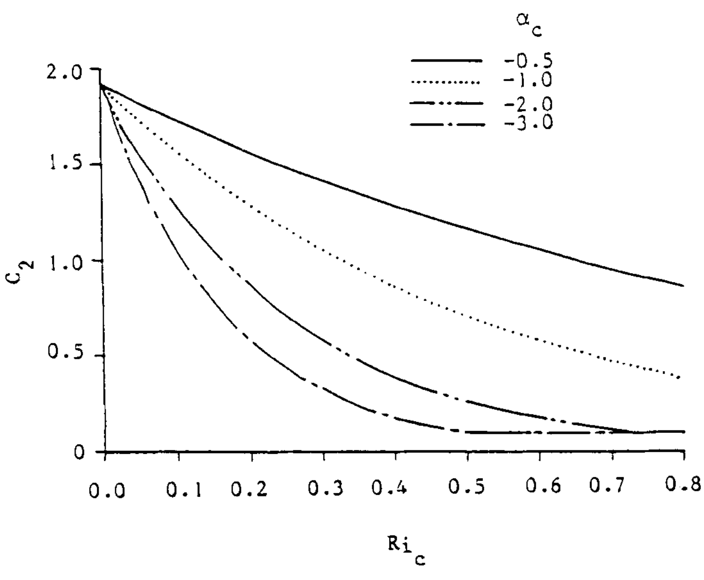

Figure 4. Changes of C_2 versus Ri_c at four values of α_c

measurements reveals that improvements using the modified k–ε model of Srinivasan and Mongia only appear in the upstream regions (i.e. recirculation zone) where Ri_c is relatively large. This observation was also reported by Srinivasan and Mongia.[4] However, in the downstream regions of the flow field the predicted mean axial velocities using the modified k–ε model are much worse

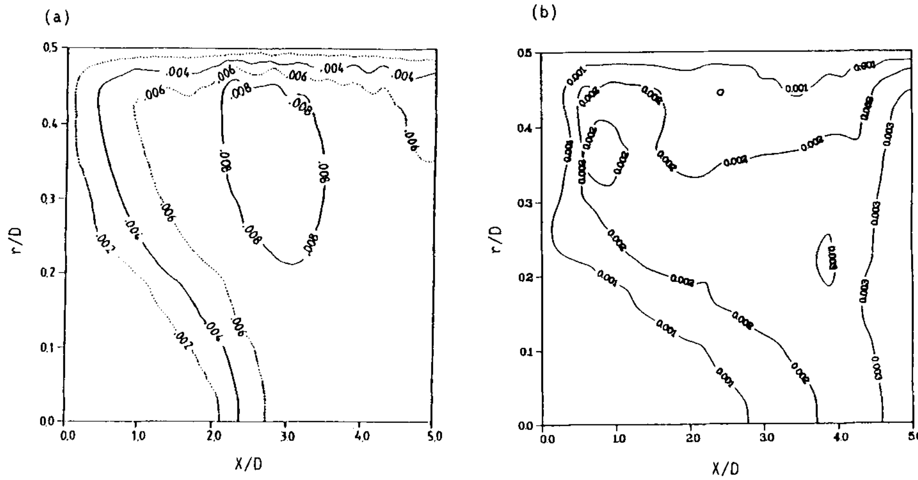

Figure 5. Comparison of contours of turbulent viscosity predicted by (a) standard k–ε model and (b) modified k–ε model[4] in test problem 1

than those using the standard k–ε model. This difference can be further highlighted by comparing the contours of turbulent viscosity obtained with both k–ε models, as shown in Figure 5. The turbulent viscosities predicted by the modified k–ε model are smaller than those by the standard k–ε model, particularly in the downstream regions, and this leads to the slow momentum diffusion rate along the lateral direction as observed in Figure 2(a). The incorrect predictions of the turbulent viscosity distribution would significantly affect the diffusion coefficients of Table I and thus result in inaccurate predictions of transport processes in the problem. From this point of view, the modification of Srinivasan and Mongia is not capable of providing satisfactory predictions of physical phenomena such as momentum, heat and mass transfers in recirculating flows.

4.2. Modification by dividing into high- and low-streamline-curvature regions

It is suggested that the modification made by Srinivasan and Mongia might overcorrect the C_2-values in areas where Ri_c is small, i.e. areas similar to the flow fields of simple flows. This idea can be supported by the fact that the empirical coefficient $C_2 = 1{\cdot}92$ was determined by the best comparison with experimental observations for most of the simple flow cases. In order to examine this idea further, a modification to the approach of Srinivasan and Mongia was made which we call Modification 1.

Modification 1. The sudden-expansion duct is split into two computational subdomains as demonstrated in Figure 1. One subdomain (the shaded region) with relatively high streamline curvature is still simulated by means of the modified k–ε model. On the other hand, for those regions in which the streamline curvature is small the calculation is done with the standard k–ε model. These low-streamline-curvature regions (see Figure 3) include two parts: one is located downstream where the recirculation zone has disappeared; the other is within the potential core. Here the potential core is defined as $u/u_{in} \geqslant 0{\cdot}99$. The predicted distributions of the mean axial velocity and the turbulent kinetic energy at various sections are presented in Figure 6. A comparison of the results presented in Figure 6 and those presented in Figure 2 indicates that Modification 1 can sustain the advantage of the modification method proposed by Srinivasan and

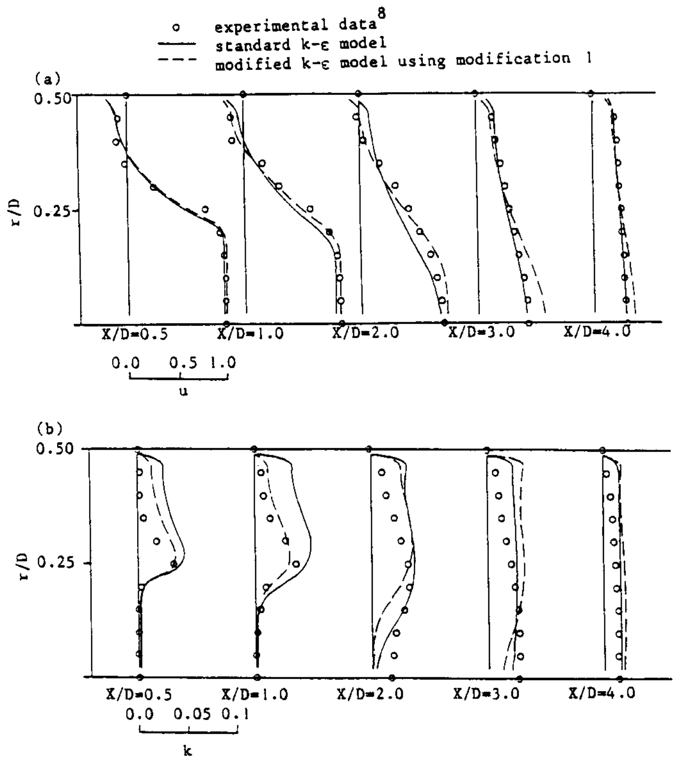

Figure 6. Comparison of predicted and measured[8] (a) axial velocities and (b) turbulent kinetic energy in test problem 1

Mongia in the recirculation zone while yielding satisfactory predictions in the downstream regions. The success of Modification 1 leads to the following attempts.

4.3. Modification using hybrid method

Next we intend to describe the approach of Modification 1 in a more quantitative manner. A hybrid k–ε model is developed to fulfil this purpose which we call Modification 2.

Modification 2. The correction formula of C_2, equation (2), is used when Ri_c is larger than a certain value, whereas $C_2 = 1{\cdot}92$ (the standard k–ε model) is used when Ri_c is smaller than this value. A number of attempts were made to find the best comparison with measurement. The following was found to yield satisfactory predictions.

$$
\begin{aligned}
C_2 &= \text{equation (2)} \quad &&\text{for } Ri_c \geqslant 0{\cdot}2,\\
C_2 &= 1{\cdot}92 &&\text{for } Ri_c < 0{\cdot}2.
\end{aligned} \tag{8}
$$

Figure 7 shows the predicted results using the modification described in equation (8). It can be observed from Figure 7 that this proposed modification indeed improves the flow field predictions in the entire computational domain. However, the price to be paid for using the hybrid k–ε model is approximately 50% more computer effort required to attain the same convergent level in comparison with that required by the standard k–ε model, and approximately 10% more computer effort than that required by the modified k–ε model of Srinivasan and Mongia.

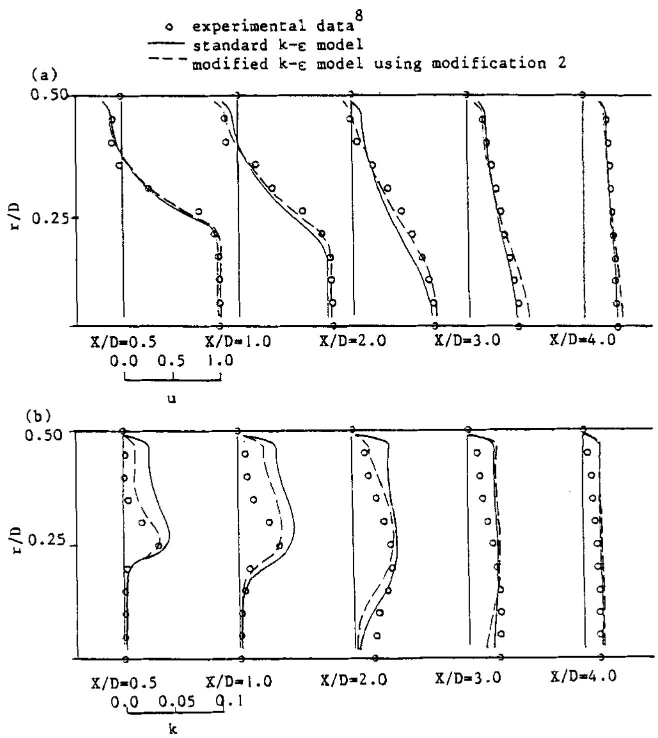

Figure 7. Comparison of predicted and measured[8] (a) axial velocities and (b) turbulent kinetic energy in test problem 1

4.4. *Comparison of reattachment lengths obtained by various models*

The predicted reattachment lengths are summarized in Table II and compared with the measurement of Chaturvedi.[8] The uncertainty in the measured reattachment length which can be ascribed to the flow unsteadiness, measurement error and inlet flow quality control[9, 10] is reported to be as high as 20%. In view of the satisfactory prediction of reattachment length (approximately 10% error) obtained with the hybrid k–ε model (i.e. Modification 2), this fact provides further confidence in the proposed hybrid modification. However, it is believed that the accuracy of the predicted reattachment length could be further improved if the more sophisticated near-wall models[1] were employed for calculations.

4.5. *Further validation of hybrid k–ε model*

Two more test problems were run in order to examine the generality of the developed hybrid model. The experimental study of an axisymmetrical sudden-expansion flow conducted by Durrett *et al.*[11] was selected as the second test problem. The dimensions of the configuration are

Table II. Comparison of reattachment lengths

Experiment	Standard k–ε model	Original modification	Modification 1	Modification 2
2·30D	2·23D	4·24D	2·80D	2·54D

$L = 660{\cdot}8$ mm, $r_a = 25$ mm and $R = 47{\cdot}6$ mm (i.e. EPR = 1·904). The Reynolds number based upon the inlet diameter is equal to $8{\cdot}4 \times 10^4$. The mean and fluctuating quantities of velocity were measured using LDV. The mean velocity profile at the inlet plane (obtained from Pitot tube measurements) was very flat. Note that the geometric shape (EPR) and the flow condition (*Re*) of this test problem are similar to those of the previous one.

Two predicted results using the proposed hybrid model and the standard model respectively are plotted in Figure 8 and compared with the measurements of Durrett *et al.*[11] Both k–ε models exhibit an insignificant difference in the prediction of mean axial velocity, but the hybrid model yields more satisfactory predictions in the recirculation zone compared to the standard model, as shown in Figure 8. The reattachment was found to occur at $x/D \simeq 2{\cdot}0$, while the predicted lengths were 1·9 and 2·3 using the standard and hybrid models respectively, and both predictions are within the uncertainty of the measured reattachment location. Since Durrett *et al.* did not measure the tangential turbulent intensity, the experimental data of turbulent kinetic energy, which is defined as $(\overline{u'^2} + \overline{v'^2} + \overline{w'^2})/2$, are not available. However, the measured axial and radial turbulent intensities were utilized for comparing the superiority of these two k–ε models. Figure 9 compares the predicted axial and radial turbulent intensities using the two k–ε models with the measurements. Clearly, the hybrid model improves the predictions in the recirculation zone.

The third test problem is for the axisymmetric sudden-expansion flow experiment of So and Ahmed[12] with dimensions $L = 508$ mm, $r_a = 21{\cdot}6$ mm and $R = 31{\cdot}75$ mm (i.e. EPR = 1·47). The velocity measurements were made by LDV and the inlet Reynolds number is $4{\cdot}6 \times 10^4$. Two velocity components along the axial and tangential directions were measured in their work. Calculations using the two different k–ε models and measurements of u and u' are given in Figure 10. In the plot of mean axial velocity (see Figure 10(a)) there exist significant discrepancies between the predicted and measured profiles in the downstream region. A check for mass conservation at each section of measurement reveals that the measured mean axial velocity profiles at $x/D = 1{\cdot}5$ and $2{\cdot}7$ yield mass flow rates which are about 12% and 45% respectively higher than that at the inlet. Nevertheless, the requirement for mass conservation in the upstream region is acceptable. However, Figure 10(a) shows that the hybrid model yields better predictions in the recirculation zone in comparison with the standard model, though the improvements are not as marked as in the previous two test problems. This can be explained by investigating the distribution contour of the curvature Richardson number of this flow. It is found that only a small region where $Ri_c \geqslant 0{\cdot}2$, comparing with Figure 3 (the case with EPR = 2·0), needs to be corrected in

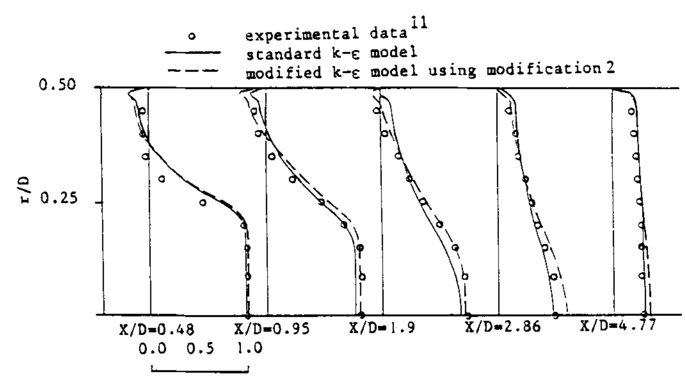

Figure 8. Comparison of predicted and measured[11] axial velocities in test problem 2

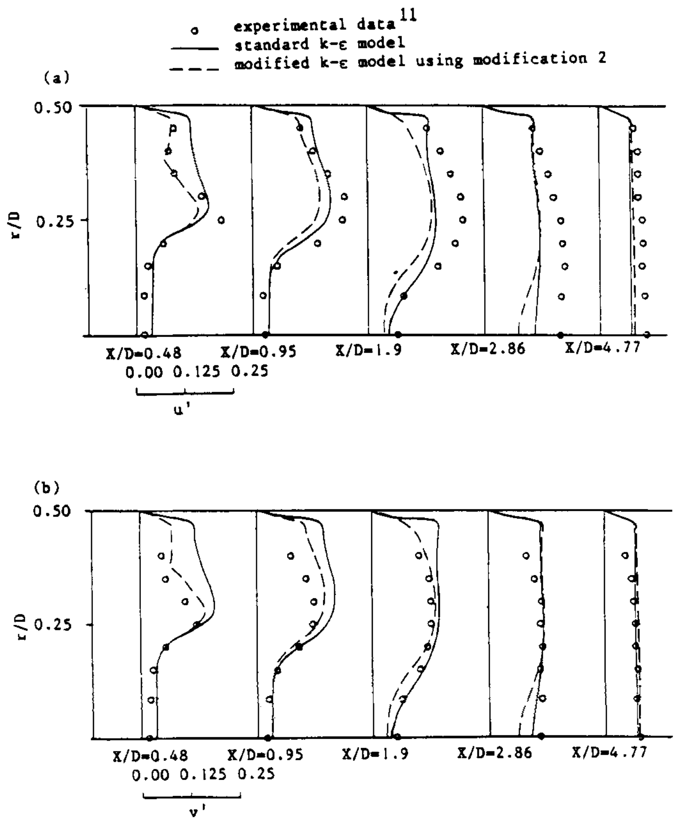

Figure 9. Comparison of predicted and measured[11] (a) axial and (b) radial turbulent intensities in test problem 2

accordance with equation (8). This leads to minor differences occuring between the predictions obtained with these two models.

A comparison of the reattachment lengths reveals that the prediction of the standard k–ε model is consistently underpredicted (1·0D versus the measurement of 1·2D), while the prediction of the hybrid model is overpredicted (1·4D) but still within the uncertainty allowance. The u'-predictions using the two k–ε models are underpredicted, particularly in the downstream region, as shown in Figure 10(b). Similar results for the u'-predictions were also obtained by Yoo and So[13] using the full Reynolds stress model.

5. CONCLUSIONS

Modifications of streamline curvature effects in the k–ε model have been examined and compared with corresponding experimental results found in the literature. It was found that the primary drawback of the modification made by Srinivasan and Mongia stemmed from the overcorrection of C_2-values in regions where the streamline curvatures are relatively small. The values of the turbulent viscosity are significantly underpredicted, and this will, in turn, result in wrong predictions of all transport processes.

The suggestion was made that for regions in which the streamline curvature is small, the modification proposed by Srinivasan and Mongia should not be employed for simulation. Based

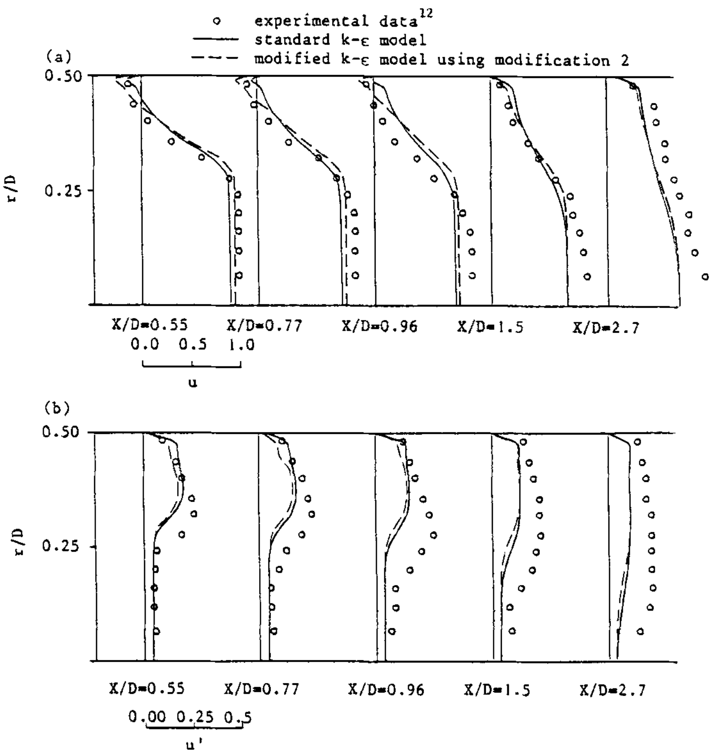

Figure 10. Comparison of predicted and measured[12] (a) axial velocities and (b) axial turbulent intensity in test problem 3

upon this idea, a hybrid k–ε model was developed and expressed as

$$C_2 = 1{\cdot}92\ \exp(2\alpha_c Ri_c) \quad \text{for } Ri_c \geqslant 0{\cdot}2,$$

$$C_2 = 1{\cdot}92 \quad \text{for } Ri_c < 0{\cdot}2,$$

where α_c was determined to be -2 in the study. This hybrid model is shown to improve the prediction accuracy of recirculating flows in comparison with the standard k–ε model and the modified model proposed by Srinivasan and Mongia. However, the application of this hybrid model to calculations of swirling flows needs to be further investigated.

APPENDIX I. WALL FUNCTION TREATMENT

The generation term G_k shown in Table I at the grid node P (see Figure 11) is calculated by

$$G_k = \tau_w u_s / \Delta r_p. \tag{9}$$

The determination of the shear stress at the wall depends upon the location of the grid node P, i.e.

$$\tau_w = 0{\cdot}42 r^+ / \ln(9{\cdot}8 r^+) \quad \text{for } r^+ > 15 \tag{10}$$

$$\tau_w = \mu u_p / r_p \quad \text{for } r^+ \leqslant 15, \tag{11}$$

where

$$r^+ = \rho C_\mu^{0{\cdot}25} k_p^{0{\cdot}5} / \mu. \tag{12}$$

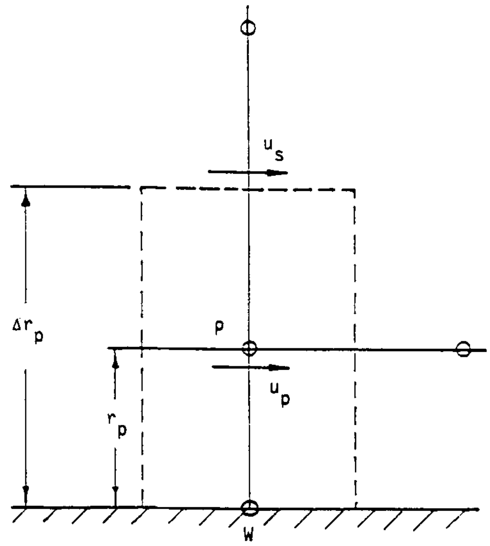

Figure 11. Near-wall grid node

APPENDIX II. NOMENCLATURE

C_1, C_2, C_μ	turbulence coefficients
D	diameter of outer duct
G_k	volumetric rate of generation for k
k	turbulent kinetic energy
L	length of sudden-expansion duct
p	pressure
P	near-wall grid node
r	radial co-ordinate
r_a	radius of inner duct
R	radius of outer duct
Ri	Richardson number
S_ϕ	source term
u, v	mean velocity components corresponding to axial and radial co-ordinates respectively
W	grid node at wall
x	axial co-ordinate
α	empirical coefficient
Γ	transport coefficient
ε	dissipation rate of turbulent kinetic energy
μ	viscosity
ρ	density
σ	turbulent diffusion coefficient
τ_w	shear stress at wall
ϕ	dependent variable

Subscripts

c	curvature
eff	effective value
in	inlet
k	turbulent kinetic energy
s	swirl
ε	dissipation rate of tubulent kinetic energy

Superscripts

$'$	fluctuating quantity
$+$	dimensionless form

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